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APPROXIMATE CORRECTIONS FOR THE EFFECTS OF COMPRESSIBILITY ON THE SUBSONIC STABILITY DERIVATIVES OF SWEPT WINGS

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SUMMARY

A method which has previously been presented for the estimation of the low-speed stability derivatives of swept wings is modified by means of the Prandtl-Glauert rule to yield approximate corrections for the first-order three-dimensional effects of compressible flow in the subsonic region. Corrections are presented in chart form for several aspect ratios and angles of sweep and may be computed rather simply for other aspect ratios and angles of sweep.

Lift-curve slopes and values of damping in roll are compared with those obtained by use of a generally accepted method and are discussed in relation to unpublished data.

INTRODUCTION

The three-dimensional effects of compressibility on the lift-curve slopes of unswept wings are treated in references 1 and 2, and the treatment is extended to the slopes of swept wings in references 3 and 4. The only other stability derivative that appears to have been considered is the damping in roll, for which compressible-flow values can be estimated from reference 5.

At present no rigorous theories exist that seem suitable for the evaluation of all the stability derivatives of swept wings even in incompressible flow. The effects of compressibility on these derivatives may not be treated in an exact manner for some time. For many applications, however, such as in dynamic stability calculations, an approximation of the effects of compressibility is adequate.

A complete analysis, based on strip theory modified by simple means to account for the primary effects of aerodynamic induction, has been presented for the contribution of sweep to the stability derivatives of wings for zero Mach number. (See reference 6.)

The method forms a convenient basis for making estimates, in a rational manner, of compressibility effects on the stability derivatives. This estimate is accomplished, in the present paper, by increasing the lift-curve slope of each wing section by an application of the Prandtl-Glauert rule and by using the equations of reference 6 to derive corrections for the first-order three-dimensional effects of compressibility in the subsonic Mach number region. The corrections are applied in the form of ratios of the compressible-flow equations to the incompressible-flow equations; thus, the effects of errors in the theory are minimized.

The same procedure is applied to all the steady-state derivatives with the exception of the yawing moment due to yawing and the lateral force due to yawing. Since the contribution of a wing alone to either of these derivatives is small compared to that of the vertical tail, Mach number effects on these derivatives would appear to be insignificant.

Consideration of the lift-curve slope and the damping in roll is included herein for completeness, although more rigorous procedures for treating these derivatives are discussed in references 1 to 5.

SYMBOLS

The symbols used in the analysis and in the presentation of results are defined herein. All spans and chords are measured perpendicular and parallel, respectively, to the plane of symmetry.

C_L lift coefficient $\left(\frac{\text{Lift}}{\frac{1}{2}\rho V^2 S}\right)$

C_l rolling-moment coefficient $\left(\frac{\text{Rolling moment}}{\frac{1}{2}\rho V^2 Sb}\right)$

C_m pitching-moment coefficient $\left(\frac{\text{Pitching moment}}{\frac{1}{2}\rho V^2 S\bar{c}}\right)$

C_n yawing-moment coefficient $\left(\frac{\text{Yawing moment}}{\frac{1}{2}\rho V^2 Sb}\right)$

C_Y lateral-force coefficient $\left(\frac{\text{Lateral force}}{\frac{1}{2}\rho V^2 S}\right)$

c_l	section primary-force coefficient (see reference 6) $\left(\dfrac{\text{Section primary force}}{\frac{1}{2}\rho V^2 c}\right)$
ρ	mass density, slugs per cubic foot
S	wing area, square feet
V	free-stream velocity, feet per second
b	wing span, feet
c	wing chord, feet
$\bar{c}$	wing mean chord, feet (S/b)
$\bar{x}$	longitudinal distance rearward from airplane center of gravity to wing aerodynamic center, feet
y	spanwise distance from plane of symmetry to any station on wing quarter-chord line, feet
$\bar{y}$	spanwise distance from plane of symmetry to effective lateral center-of-pressure location of resulting load causing rolling moment, feet
A	aspect ratio (b^2/S)
λ	taper ratio $\left(\dfrac{\text{Tip chord}}{\text{Root chord}}\right)$
M	Mach number $\left(\dfrac{V}{\text{Speed of sound}}\right)$
Λ	angle of sweep measured at quarter-chord line, degrees
α	angle of attack, radians
β	angle of sideslip, radians
β'	local angle of sideslip; angle between plane of symmetry and local air-stream direction at quarter-chord point of any section, radians

$$B_o = \sqrt{1 - M^2}$$

$$B = \sqrt{1 - M^2\cos^2\Lambda}$$

a_o section lift-curve slope for section normal to quarter-chord line when placed in direction of free stream

p rolling angular velocity, radians per second

q pitching angular velocity, radians per second

r yawing angular velocity, radians per second

$\frac{pb}{2V}$ nondimensional rolling-velocity parameter

$\frac{qc}{2V}$ nondimensional pitching-velocity parameter

$\frac{rb}{2V}$ nondimensional yawing-velocity parameter

Subscripts:

L left wing panel

R right wing panel

M value at Mach number M

$\Lambda=0^{\circ}$ restricted to zero sweep

α,β,p,r,q denotes derivative of C_L, C_m, C_l, C_n, or C_Y with respect to α, β, $\frac{pb}{2V}$, $\frac{rb}{2V}$, and $\frac{qc}{2V}$; for example, $C_{l_\beta} \equiv \frac{\partial C_l}{\partial \beta}$ and $C_{n_p} \equiv \frac{\partial C_n}{\partial\left(\frac{pb}{2V}\right)}$

ANALYSIS

In reference 1, three variations of the application of the theory of small perturbations to determine the effects of compressibility are summarized. Two of these methods consist of changing the dimensions of the wing or the angle of attack or both through the Prandtl-Glauert transformation and then obtaining the aerodynamic characteristics of the resulting wing from incompressible-flow theory. These concepts are disadvantageous in that the plan form of the wing in sideslip or yawing is

distorted unsymmetrically. The third method, which consists of increasing the two-dimensional lift-curve slope of a wing for which the dimensions were held constant, is the most convenient to use and is the procedure adopted herein.

Reference 1 shows that the increase in section lift-curve slope is the only first-order three-dimensional effect of compressibility on a thin wing. For an unswept and untapered wing in a free-stream flow of Mach number M, the section lift-curve slope a_o is shown to be increased by the Prandtl-Glauert factor $1/B_o$ where $B_o = \sqrt{1 - M^2}$. If the wing is swept or oblique to the flow, however, only the component of the Mach number in the direction normal to the quarter-chord line is effective in increasing the section lift-curve slope of the wing. (See reference 7.) Thus the lift of a thin two-dimensional wing which is oblique to the flow by an angle Λ is increased by the factor $1/B$ where $B = \sqrt{1 - M^2\cos^2\Lambda}$. A somewhat greater rate of increase of wing lift with Mach number would be expected for wings of finite thickness (reference 8), but it is shown in reference 4 that this effect is small for usual aspect ratios and sweep angles. For the present analysis, therefore, the effects of thickness will be neglected.

The stability derivatives under consideration in this analysis depend substantially upon the orientation and the rate of change of lift and induced-drag (which depends on lift) forces due to a change in attitude of the wing. When these lift forces are increased in accordance with the Prandtl-Glauert rule, the stability derivatives that depend so directly upon the lift forces may be expected to include the first-order effects of compressibility. The equations of reference 6 consider the two-dimensional effects of sweep corrected for finite aspect ratios. If the two-dimensional effects are further corrected for compressibility, the resulting equations should also be valid for three-dimensional flow.

In reference 6, strip-theory equations were first derived for each of the derivatives with approximate consideration given to aerodynamic induction. Since the strip theory would be quantitatively in error, these equations were used only to obtain corrections to unswept-wing derivatives, which could be calculated by more exact theories.

In the derivations presented in reference 6, the section lift-curve slope was left arbitrary only when it had a primary effect on the derivative; that is, when the derivative was proportional to a_o. In other instances a_o was assumed equal to 2π. For the determination of the compressibility-correction equations in the present paper, the strip-theory equations of reference 6 are restated in more basic form by retaining a_o in all instances. The compressible-flow equation is established by substituting a_o in the incompressible-flow development with a_o/B and then by replacing a_o with 2π. In the case of sideslipping or yawing, the problem seems a little more complex unless it is understood that the local

conditions at each wing section are considered. The Mach number correction is then the ratio of the compressible-flow equation to its incompressible-flow counterpart. This ratio, as it varies with Mach number, may readily be applied as a correction to incompressible-flow values of a derivative which may be determined from reference 6 or from experimental data. The derivations for the Mach number corrections for all stability derivatives considered are given in the appendix. Compressibility effects on the derivatives C_{n_r} and C_{Y_r} are not considered in this paper, although a logical method for the estimation of the correction to C_{n_r} is suggested in the appendix.

The present method is subject to the limitations of the linear perturbation theory that assumes only small departures of the fluid velocity from the free-stream velocity. The strip theory of reference 6 is limited for most accurate results to high aspect ratios and taper ratios close to unity. Errors inherent in the equations, such as those magnified by low aspect ratios, however, tend to nullify each other when the correction is applied in the form of a ratio. For the same reason the equations are considered to be applicable to wings that are tapered at least moderately.

RESULTS AND DISCUSSION

The correction factors derived for some of the derivatives depend on the static margin $\bar{x}/\bar{c}$. In order to provide an indication of the importance of the static margin, the factors dependent upon it were calculated for static margins of 0 and 0.2 for wings having sweep angles of 30° and 50° and an aspect ratio of 4. The results shown in figure 1 indicate that the static margin is of little importance; consequently, the margin was assumed to be zero for those equations in which it appeared.

The corrections which may be multiplied by the incompressible-flow values of the stability derivatives under consideration in order to arrive at approximate values for the derivatives at Mach numbers between zero and unity are presented in chart form (figs. 2 to 9). The charts present corrections for four aspect ratios (2, 3, 4, and 6) and five angles of sweep (0°, 30°, 40°, 50°, and 60°). Compressibility corrections for aspect ratios and sweep angles other than for those presented may be obtained either by interpolation of the given curves or by calculation from the given equations. (See table I.) These corrections are applicable to either sweptback or sweptforward wings within the limitations of the basic theory.

The calculated effects of compressibility on the stability derivatives of representative wings having an aspect ratio of 4 are presented in figure 10, which shows that an increase in sweep may be expected to decrease the magnitude of the increments due to compressibility. Values of the derivatives for zero Mach number were obtained from reference 6 and any effects of static margin were neglected.

A comparison of the present method with the generally accepted method of equivalent wings for determining approximate Mach number effects on C_{L_α} and C_{l_p} is given in figures 11 to 14. The use of the equivalent-wing method in conjunction with calculations made by the Weissinger method is suggested in references 3 and 5 for C_{L_α} and C_{l_p}, respectively. A preliminary comparison of these methods with some unpublished experimental data indicates that both methods underestimate the effects of compressibility on C_{L_α}, particularly for the smaller sweep angles and higher Mach numbers. This underestimation is probably due in part to the neglect of the finite-thickness effect of the experimental wings. The present method indicates greater variations with Mach number than the equivalent-wing method (see figs. 11 to 14) and, therefore, is expected to give better agreement with experimental results.

The present method was derived by application of a form of strip theory to wings having a taper ratio of 1.0. An indication of the reliability of the method for moderately tapered wings is provided for the derivatives C_{L_α} and C_{l_p} in figures 11 and 13, respectively. These figures show that for an aspect ratio of 4 the differences between the present method and the method of equivalent wings are about the same for taper ratios of either 1.0 or 0.5. The present method, therefore, might be expected to apply with reasonable accuracy to moderately tapered wings. Comparisons of the present method with the equivalent-wing method for untapered wings having aspect ratios of 2 and 6 show that, over this range of aspect ratios, the present method yields consistently larger values of the corrections for either C_{L_α} or C_{l_p}. (See figs. 12 and 14.) Both methods are in general agreement in that, for a given aspect ratio, smaller effects of compressibility are indicated as the sweep angle is increased.

CONCLUDING REMARKS

An adaptation of the Prandtl-Glauert rule is used to modify existing theory for the first-order effects of compressibility on the subsonic stability derivatives of swept wings. Lift-curve slopes and values of damping in roll are compared with those obtained by use of a generally accepted method and are discussed in relation to unpublished data. Because of the lack of experimental data, the reliability of the present method for the calculation of the effect of high subsonic Mach numbers on all the stability derivatives is not known; however, the method is expected at least

to provide a reliable indication of the order of magnitude of the Mach number effect.

In general, the effects of compressibility appear to become smaller as the angle of sweep is increased.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., January 20, 1949

APPENDIX

DERIVATION OF COMPRESSIBILITY-CORRECTION EQUATIONS

Lift-Curve Slope

The strip-theory relationship for $\left(C_{L_\alpha}\right)_{M=0}$ in its basic form, as given by reference 6, is

$$\left(C_{L_\alpha}\right)_{M=0} = \frac{a_o \cos \Lambda}{1 + \frac{a_o \cos \Lambda}{\pi A}}$$

The application of the Prandtl-Glauert rule gives the result

$$\left(C_{L_\alpha}\right)_M = \frac{\frac{a_o}{B} \cos \Lambda}{1 + \frac{a_o}{B} \frac{\cos \Lambda}{\pi A}}$$

If the thin-airfoil-theory value of 2π is now assigned to a_o, the compressibility-correction equation becomes

$$\frac{\left(C_{L_\alpha}\right)_M}{\left(C_{L_\alpha}\right)_{M=0}} = \frac{A + 2 \cos \Lambda}{AB + 2 \cos \Lambda} \tag{1}$$

At zero sweep angle, equation (1) reduces to the equation derived in reference 2 by increasing the lift coefficient by $1/B^2$ and decreasing the span and angle of attack by B in the lifting-line-theory equation for C_{L_α}.

Equation (1) indicates an increase in the compressibility factor from unity to $1/B$ as the aspect ratio increases from zero to infinity.

Rolling Flight

Rolling moment due to rolling.- A procedure similar to that employed for C_{L_α} can be used to develop an expression for the effects of compressibility on the damping in roll C_{l_p} based on the following equation from reference 6 in basic form:

$$\left(C_{l_p}\right)_{M=0} = -\frac{1}{6} \frac{Aa_o \cos \Lambda}{A + \frac{2a_o \cos \Lambda}{\pi}}$$

The application of the Prandtl-Glauert rule gives

$$\left(C_{l_p}\right)_M = -\frac{1}{6}\frac{A\frac{a_o}{B}\cos\Lambda}{A + \frac{a_o}{B}\frac{2\cos\Lambda}{\pi}}$$

If $a_o = 2\pi$, then the compressibility-correction equation is

$$\frac{\left(C_{l_p}\right)_M}{\left(C_{l_p}\right)_{M=0}} = \frac{A + 4\cos\Lambda}{AB + 4\cos\Lambda} \tag{2}$$

Yawing moment due to rolling.- The incompressible-flow relationship for C_{n_p}, based on reference 6, is

$$\left(C_{n_p}\right)_{M=0} = -\frac{C_L}{6}\left[\frac{A + 6\left(A + \frac{a_o\cos\Lambda}{2\pi}\right)\left(\frac{\bar{x}}{\bar{c}}\frac{\tan\Lambda}{A} + \frac{\tan^2\Lambda}{12}\right)}{A + \frac{2a_o\cos\Lambda}{\pi}}\right]$$

When modified for compressibility, the equation becomes

$$\left(C_{n_p}\right)_M = -\frac{C_L}{6}\left[\frac{A + 6\left(A + \frac{a_o}{B}\frac{\cos\Lambda}{2\pi}\right)\left(\frac{\bar{x}}{\bar{c}}\frac{\tan\Lambda}{A} + \frac{\tan^2\Lambda}{12}\right)}{A + \frac{2\cos\Lambda}{\pi}\frac{a_o}{B}}\right]$$

By letting $a_o = 2\pi$, the compressibility-correction equation becomes

$$\frac{\left(\frac{C_{n_p}}{C_L}\right)_M}{\left(\frac{C_{n_p}}{C_L}\right)_{M=0}} = \frac{A + 4\cos\Lambda}{AB + 4\cos\Lambda}\;\frac{AB + 6\left(AB + \cos\Lambda\right)\left(\frac{\bar{x}}{\bar{c}}\frac{\tan\Lambda}{A} + \frac{\tan^2\Lambda}{12}\right)}{A + 6\left(A + \cos\Lambda\right)\left(\frac{\bar{x}}{\bar{c}}\frac{\tan\Lambda}{A} + \frac{\tan^2\Lambda}{12}\right)}$$

Figure 1 shows that the static margin $\bar{x}/\bar{c}$ has a negligible effect on any correction equation in which it appears. If $\bar{x}/\bar{c}$ is restricted to zero, the compressibility-correction equation for the yawing moment due to rolling becomes

$$\frac{\left(\frac{C_{n_p}}{C_L}\right)_M}{\left(\frac{C_{n_p}}{C_L}\right)_{M=0}} = \frac{A + 4 \cos \Lambda}{AB + 4 \cos \Lambda} \frac{AB + \frac{1}{2}\left(AB + \cos \Lambda\right)\tan^2\Lambda}{A + \frac{1}{2}\left(A + \cos \Lambda\right)\tan^2\Lambda} \tag{3}$$

Lateral force due to rolling.- The equation for C_{Y_p}, as developed for the incompressible-flow analysis of reference 6, is

$$\left(C_{Y_p}\right)_{M=0} = C_L \tan \Lambda \frac{A + \frac{a_o \cos \Lambda}{2\pi}}{A + \frac{2a_o \cos \Lambda}{\pi}}$$

The value for the derivative at Mach number M may be written

$$\left(C_{Y_p}\right)_M = C_L \tan \Lambda \frac{A + \frac{a_o}{B} \frac{\cos \Lambda}{2\pi}}{A + \frac{a_o}{B} \frac{2 \cos \Lambda}{\pi}}$$

If $a_o = 2\pi$, then

$$\frac{\left(\frac{C_{Y_p}}{C_L}\right)_M}{\left(\frac{C_{Y_p}}{C_L}\right)_{M=0}} = \frac{A + 4 \cos \Lambda}{AB + 4 \cos \Lambda} \frac{AB + \cos \Lambda}{A + \cos \Lambda} \tag{4}$$

Sideslipping Flight

Rolling moment due to sideslip.- In the case of a swept wing in sideslip, the different effective sweep angles produce a difference in effective Mach number between the left and the right wing panels. Calculations, however, show this difference in Mach number to be of second-order importance. The effect may, therefore, be neglected and a procedure similar to that employed heretofore may again be used.

The equation for C_{l_β}, as developed for incompressible flow in reference 6, is

$$\left(\frac{C_{l_\beta}}{C_L}\right)_{M=0} = \left(\frac{C_{l_\beta}}{C_L}\right)_{\Lambda=0^\circ} - \frac{\bar{y}}{b/2} \frac{A + \dfrac{a_o \cos \Lambda}{\pi}}{A + \dfrac{2a_o \cos \Lambda}{\pi}} \frac{\tan \Lambda}{2} \tag{5}$$

The procedure employed in the present paper is actually applicable only to the increment in $\left(\frac{C_{l_\beta}}{C_L}\right)$ that is due to sweep. Although the other increment cannot be handled by the present simplified procedure, it probably is affected by Mach number in much the same manner as the increment due to sweep, since both increments result from load distributions of the same general form. The compressibility factor determined for the increment due to sweep, therefore, is assumed to apply to the total value of the derivative. The additional assumption is made that compressibility causes no material change in the lateral center of pressure.

If $\left(\frac{C_{l_\beta}}{C_L}\right)_{\Lambda=0^\circ}$ is neglected and the Prandtl-Glauert rule is applied to the increment due to sweep, the equation becomes

$$\left(\frac{C_{l_\beta}}{C_L}\right)_M = -\frac{\bar{y}}{b/2} \frac{A + \dfrac{a_o}{B} \dfrac{\cos \Lambda}{\pi}}{A + \dfrac{a_o}{B} \dfrac{2 \cos \Lambda}{\pi}} \frac{\tan \Lambda}{2}$$

If the thin-airfoil-theory value of 2π is substituted for a_o, the compressibility factor becomes

$$\frac{\left(\frac{C_{l_\beta}}{C_L}\right)_M}{\left(\frac{C_{l_\beta}}{C_L}\right)_{M=0}} = \frac{A + 4 \cos \Lambda}{AB + 4 \cos \Lambda} \frac{AB + 2 \cos \Lambda}{A + 2 \cos \Lambda} \tag{6}$$

Equation (6) should now be employed in the following manner to correct $\left(C_{l_\beta}\right)_{M=0}$ as calculated from equation (5):

$$\left(\frac{C_{l_\beta}}{C_L}\right)_M = \left[\left(\frac{C_{l_\beta}}{C_L}\right)_{\Lambda=0^\circ} - \frac{\bar{y}}{b/2} \frac{A + \dfrac{a_o \cos \Lambda}{\pi}}{A + \dfrac{2a_o \cos \Lambda}{\pi}} \frac{\tan \Lambda}{2}\right] \frac{A + 4 \cos \Lambda}{AB + 4 \cos \Lambda} \frac{AB + 2 \cos \Lambda}{A + 2 \cos \Lambda}$$

where $\left(\frac{C_{l_\beta}}{C_L}\right)_{\Lambda=0^\circ}$ and $\frac{\bar{y}}{b/2}$ may be determined from reference 6.

Yawing moment due to sideslip.- The equation for $\left(C_{n_\beta}\right)_{M=0}$, as developed in reference 6, is

$$\left(\frac{C_{n_\beta}}{C_L^2}\right)_{M=0} = \left(\frac{C_{n_\beta}}{C_L^2}\right)_{\Lambda=0^\circ} - \frac{\tan \Lambda}{\pi A\left(A + \frac{2a_o \cos \Lambda}{\pi}\right)}\left(3\frac{\bar{x}}{\bar{c}}\frac{a_o \sin \Lambda}{\pi A} + \frac{a_o \cos \Lambda}{2\pi} - \frac{A}{2} - \frac{A^2 \pi}{4a_o \cos \Lambda}\right) \tag{7}$$

If $\left(\frac{C_{n_\beta}}{C_L^2}\right)_{\Lambda=0^\circ}$ is neglected and the Prandtl-Glauert rule is applied to the increment due to sweep, the equation becomes

$$\left(\frac{C_{n_\beta}}{C_L^2}\right)_M = -\frac{\tan \Lambda}{\pi A\left(A + \frac{a_o}{B}\frac{2 \cos \Lambda}{\pi}\right)}\left(3\frac{\bar{x}}{\bar{c}}\frac{\sin \Lambda}{\pi A} + \frac{a_o}{B}\frac{\cos \Lambda}{2\pi} - \frac{A}{2} - \frac{A^2\pi}{4\frac{a_o}{B}\cos \Lambda}\right)$$

By letting $a_o = 2\pi$, the compressibility factor is

$$\frac{\left(\frac{C_{n_\beta}}{C_L^2}\right)_M}{\left(\frac{C_{n_\beta}}{C_L^2}\right)_{M=0}} = \frac{A + 4 \cos \Lambda}{AB + 4 \cos \Lambda}\;\frac{6\frac{\bar{x}}{\bar{c}}\frac{\sin \Lambda}{A} + \cos \Lambda - \frac{AB}{2} - \frac{A^2B^2}{8 \cos \Lambda}}{6\frac{\bar{x}}{\bar{c}}\frac{\sin \Lambda}{A} + \cos \Lambda - \frac{A}{2} - \frac{A^2}{8 \cos \Lambda}}$$

If $\bar{x}/\bar{c}$ is restricted to zero, the equation becomes

$$\frac{\left(\frac{C_{n_\beta}}{C_L^2}\right)_M}{\left(\frac{C_{n_\beta}}{C_L^2}\right)_{M=0}} = \frac{A + 4 \cos \Lambda}{AB + 4 \cos \Lambda} \frac{A^2B^2 + 4AB \cos \Lambda - 8 \cos^2\Lambda}{A^2 + 4A \cos \Lambda - 8 \cos^2\Lambda} \tag{8}$$

Equation (8) may now be used with respect to $\left(\frac{C_{n_\beta}}{C_L^2}\right)_{M=0}$ in the same manner as equation (6) was used with respect to $\left(\frac{C_{l_\beta}}{C_L}\right)_{M=0}$. The derivative $\left(\frac{C_{n_\beta}}{C_L^2}\right)_{\Lambda=0^\circ}$ may be determined from reference 9.

Lateral force due to sideslip.- The basic equation for $\left(C_{Y_\beta}\right)_{M=0}$, developed in reference 6, is

$$\left(\frac{C_{Y_\beta}}{C_L^2}\right)_{M=0} = \frac{3a_o \tan \Lambda \sin \Lambda}{\pi^2 A\left(A + \frac{2a_o \cos \Lambda}{\pi}\right)}$$

The corresponding Mach number equation becomes

$$\left(\frac{C_{Y_\beta}}{C_L^2}\right)_M = \frac{3\frac{a_o}{B} \tan \Lambda \sin \Lambda}{\pi^2 A\left(A + \frac{a_o}{B} \frac{2 \cos \Lambda}{\pi}\right)}$$

By letting $a_o = 2\pi$, the compressibility factor is

$$\frac{\left(\frac{C_{Y_\beta}}{C_L^2}\right)_M}{\left(\frac{C_{Y_\beta}}{C_L^2}\right)_{M=0}} = \frac{A + 4 \cos \Lambda}{AB + 4 \cos \Lambda} \tag{9}$$

Pitching Flight

Lift due to pitching.- Reference 6 presents the following equation for lift due to pitching:

$$\left(C_{L_q}\right)_{M=0} = \left(\frac{1}{2} + 2\frac{\bar{x}}{\bar{c}}\right)\left(C_{L_\alpha}\right)_{M=0}$$

The compressible-flow equation, therefore, is

$$\left(C_{L_q}\right)_M = \left(\frac{1}{2} + 2\frac{\bar{x}}{\bar{c}}\right)\left(C_{L_\alpha}\right)_M$$

then

$$\frac{\left(C_{L_q}\right)_M}{\left(C_{L_q}\right)_{M=0}} = \frac{\left(C_{L_\alpha}\right)_M}{\left(C_{L_\alpha}\right)_{M=0}} = \frac{A + 2 \cos \Lambda}{AB + 2 \cos \Lambda} \tag{10}$$

Pitching moment due to pitching.- In the incompressible-flow development (reference 6) the damping in pitch is presented as

$$\left(C_{m_q}\right)_{M=0} = -a_o \cos \Lambda \left\{\frac{A}{A + \frac{a_o \cos \Lambda}{\pi}}\left[2\left(\frac{\bar{x}}{\bar{c}}\right)^2 + \frac{1}{2}\frac{\bar{x}}{\bar{c}}\right] + \frac{1}{24}\frac{A^3 \tan^2 \Lambda}{A + \frac{3a_o \cos \Lambda}{\pi}} + \frac{1}{8}\right\}$$

The modification of this equation for compressibility results in

$$\left(C_{m_q}\right)_M = -\frac{a_o}{B} \cos \Lambda \left\{\frac{A}{A + \frac{a_o}{B}\frac{\cos \Lambda}{\pi}}\left[2\left(\frac{\bar{x}}{\bar{c}}\right)^2 + \frac{1}{2}\frac{\bar{x}}{\bar{c}}\right] + \frac{1}{24}\frac{A^3 \tan^2 \Lambda}{A + \frac{3 \cos \Lambda}{\pi}\frac{a_o}{B}} + \frac{1}{8}\right\}$$

Upon setting a_o equal to 2π, the compressibility-correction equation for the pitching moment due to pitching becomes

$$\frac{\left(C_{m_q}\right)_M}{\left(C_{m_q}\right)_{M=0}} = \frac{\dfrac{A}{AB + 2 \cos \Lambda}\left[2\left(\dfrac{\bar{x}}{\bar{c}}\right)^2 + \dfrac{1}{2}\dfrac{\bar{x}}{\bar{c}}\right] + \dfrac{1}{24}\dfrac{A^3 \tan^2 \Lambda}{AB + 6 \cos \Lambda} + \dfrac{1}{8B}}{\dfrac{A}{A + 2 \cos \Lambda}\left[2\left(\dfrac{\bar{x}}{\bar{c}}\right)^2 + \dfrac{1}{2}\dfrac{\bar{x}}{\bar{c}}\right] + \dfrac{1}{24}\dfrac{A^3 \tan^2 \Lambda}{A + 6 \cos \Lambda} + \dfrac{1}{8}}$$

If $\bar{x}/\bar{c}$ is restricted to zero

$$\frac{\left(C_{m_q}\right)_M}{\left(C_{m_q}\right)_{M=0}} = \frac{\dfrac{A^3 \tan^2 \Lambda}{AB + 6 \cos \Lambda} + \dfrac{3}{B}}{\dfrac{A^3 \tan^2 \Lambda}{A + 6 \cos \Lambda} + 3} \tag{11}$$

Yawing Flight

Rolling moment due to yawing.- In the determination of the effect of compressibility on the yawing derivatives of a wing, another factor is encountered which must be considered; that is, the spanwise variation in Mach number.

In reference 6, C_{l_r} is shown to be the result of two force coefficients c_{l_L} and c_{l_R} acting on the left and right wings, respectively. The following equation is presented:

$$C_l = \frac{1}{4}\int_0^1 \left(c_{l_L} - c_{l_R}\right) \frac{y}{b/2}\, d\left(\frac{y}{b/2}\right) \tag{12}$$

where in basic form

$$\left.\begin{aligned} c_{l_L} &= C_L\left(\frac{1 + \dfrac{a_o \cos \Lambda}{\pi A}}{a_o \cos \Lambda}\right)\left(\frac{a_o \cos \Lambda}{1 + \dfrac{a_o \cos \Lambda}{\pi A}} - \frac{\beta' a_o \sin \Lambda}{1 + \dfrac{2a_o \cos \Lambda}{\pi A}}\right)\left(1 + \frac{2ry}{V}\right) \\ c_{l_R} &= C_L\left(\frac{1 + \dfrac{a_o \cos \Lambda}{\pi A}}{a_o \cos \Lambda}\right)\left(\frac{a_o \cos \Lambda}{1 + \dfrac{a_o \cos \Lambda}{\pi A}} + \frac{\beta' a_o \sin \Lambda}{1 + \dfrac{2a_o \cos \Lambda}{\pi A}}\right)\left(1 - \frac{2ry}{V}\right) \end{aligned}\right\} \tag{13}$$

In equations (13), the expression in the first parenthesis is related to the wing lift-curve slope, which depends only on the free-stream Mach number; the expression in the second parenthesis is related to the local wing section lift coefficient, which varies along the span and must be modified for the spanwise variation of Mach number; the expression in the third parenthesis modifies equations (13) for the spanwise variation in velocity in yawing flow.

By letting

$$B_L = \sqrt{1 - M^2\cos^2\Lambda\left(1 + 2\frac{ry}{V}\right)}$$
$$\approx B - \frac{M^2\cos^2\Lambda\,\frac{ry}{V}}{B}$$

and

$$B_R = \sqrt{1 - M^2\cos^2\Lambda\left(1 - 2\frac{ry}{V}\right)}$$
$$\approx B + \frac{M^2\cos^2\Lambda\,\frac{ry}{V}}{B}$$

equations (13) when modified for compressibility become

$$c_{l_L} = C_L\left(\frac{1 + \frac{a_o}{B}\frac{\cos\Lambda}{\pi A}}{\frac{a_o}{B}\cos\Lambda}\right)\left(\frac{\frac{a_o}{B_L}\cos\Lambda}{1 + \frac{a_o}{B_L}\frac{\cos\Lambda}{\pi A}} - \frac{\beta'\frac{a_o}{B_L}\sin\Lambda}{1 + 2\frac{a_o}{B_L}\frac{\cos\Lambda}{\pi A}}\right)\left(1 + 2\frac{ry}{V}\right)$$

and

$$c_{l_R} = C_L\left(\frac{1 + \frac{a_o}{B}\frac{\cos\Lambda}{\pi A}}{\frac{a_o}{B}\cos\Lambda}\right)\left(\frac{\frac{a_o}{B_R}\cos\Lambda}{1 + \frac{a_o}{B_R}\frac{\cos\Lambda}{\pi A}} + \frac{\beta'\frac{a_o}{B_R}\sin\Lambda}{1 + 2\frac{a_o}{B_R}\frac{\cos\Lambda}{\pi A}}\right)\left(1 - 2\frac{ry}{V}\right)$$

where, from reference 6,

$$\beta' = -\frac{rb}{2V}\left[\frac{\bar{x}}{b/2} + \left(\frac{y}{b/2} - \frac{1}{2}\right)\tan\Lambda\right]$$

Therefore,

$$c_{l_L} - c_{l_R} = 2C_L \frac{rb}{2V} \left\{ \frac{y}{b/2} \left[2 + \frac{1 - B^2}{B\left(B + \frac{a_o \cos \Lambda}{\pi A}\right)} \right] + \tan \Lambda \frac{B + \frac{a_o \cos \Lambda}{\pi A}}{B + \frac{2a_o \cos \Lambda}{\pi A}} \left[\frac{\bar{x}}{b/2} + \left(\frac{y}{b/2} - \frac{1}{2} \right) \tan \Lambda \right] \right\} \quad (14)$$

After substituting this value of $c_{l_L} - c_{l_R}$ from equation (14) into equation (12), integrating across the span, differentiating the result with respect to the yawing-velocity parameter $\frac{rb}{2V}$, and replacing a_o by 2π, the Mach number value of the derivative of rolling moment due to yawing becomes

$$\left(\frac{C_{l_r}}{C_L}\right)_M = \frac{1}{6}\left[2 + \frac{A(1 - B^2)}{B(AB + 2 \cos \Lambda)}\right] + \frac{AB + 2 \cos \Lambda}{AB + 4 \cos \Lambda}\left(\frac{\bar{x}}{\bar{c}} \frac{\tan \Lambda}{2A} + \frac{\tan^2\Lambda}{24}\right)$$

At zero Mach number this equation reduces to the equation presented in reference 6:

$$\left(\frac{C_{l_r}}{C_L}\right)_{M=0} = \frac{1}{3} + \frac{A + 2 \cos \Lambda}{A + 4 \cos \Lambda}\left(\frac{\bar{x}}{\bar{c}} \frac{\tan \Lambda}{2A} + \frac{\tan^2\Lambda}{24}\right)$$

Therefore,

$$\frac{\left(\frac{C_{l_r}}{C_L}\right)_M}{\left(\frac{C_{l_r}}{C_L}\right)_{M=0}} = \frac{\frac{1}{6}\left[2 + \frac{A(1 - B^2)}{B(AB + 2 \cos \Lambda)}\right] + \frac{AB + 2 \cos \Lambda}{AB + 4 \cos \Lambda}\left(\frac{\bar{x}}{\bar{c}} \frac{\tan \Lambda}{2A} + \frac{\tan^2\Lambda}{24}\right)}{\frac{1}{3} + \frac{A + 2 \cos \Lambda}{A + 4 \cos \Lambda}\left(\frac{\bar{x}}{\bar{c}} \frac{\tan \Lambda}{2A} + \frac{\tan^2\Lambda}{24}\right)}$$

If $\bar{x}/\bar{c}$ is restricted to zero,

$$\frac{\left(\dfrac{C_{l_r}}{C_L}\right)_M}{\left(\dfrac{C_{l_r}}{C_L}\right)_{M=0}} = \frac{1 + \dfrac{A(1 - B^2)}{2B(AB + 2 \cos \Lambda)} + \dfrac{AB + 2 \cos \Lambda}{AB + 4 \cos \Lambda} \dfrac{\tan^2 \Lambda}{8}}{1 + \dfrac{A + 2 \cos \Lambda}{A + 4 \cos \Lambda} \dfrac{\tan^2 \Lambda}{8}} \tag{15}$$

Yawing moment due to yawing and lateral force due to yawing.- The contribution of a wing alone to either C_{n_r} or C_{Y_r} is small compared to the contribution of the vertical tail. For this reason, these two derivatives were not deemed as being of sufficient importance to warrant consideration similar to that given the derivative C_{l_r}. The derivatives may be corrected for compressibility effects on a similar basis; however, the procedure used for these derivatives could not be expected to be very reliable because the drag and the drag distribution are important but cannot be handled in a logical manner. Perhaps the most reliable procedure for the derivative C_{n_r} is the use of the incompressible-flow equation of reference 6 with the wing profile-drag coefficient appropriate to the Mach number in question.

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TABLE I.- SUMMARY OF APPROXIMATE CORRECTIONS FOR THE EFFECTS OF COMPRESSIBILITY ON STABILITY DERIVATIVES

Derivative	Relation (1)	Equation	Figure
$\left(C_{L_\alpha}\right)_M$	$\frac{A + 2\cos\Lambda}{AB + 2\cos\Lambda}\left(C_{L_\alpha}\right)_{M=0}$	(1)	2
$\left(C_{l_p}\right)_M$	$\frac{A + 4\cos\Lambda}{AB + 4\cos\Lambda}\left(C_{l_p}\right)_{M=0}$	(2)	3
$\left(\frac{C_{n_p}}{C_L}\right)_M$	$\frac{A + 4\cos\Lambda}{AB + 4\cos\Lambda}\;\frac{AB + \frac{1}{2}(AB + \cos\Lambda)\tan^2\Lambda}{A + \frac{1}{2}(A + \cos\Lambda)\tan^2\Lambda}\left(\frac{C_{n_p}}{C_L}\right)_{M=0}$	(3)	4
$\left(\frac{C_{Y_p}}{C_L}\right)_M$	$\frac{A + 4\cos\Lambda}{AB + 4\cos\Lambda}\;\frac{AB + \cos\Lambda}{A + \cos\Lambda}\left(\frac{C_{Y_p}}{C_L}\right)_{M=0}$	(4)	5
$\left(\frac{C_{l_\beta}}{C_L}\right)_M$	$\frac{A + 4\cos\Lambda}{AB + 4\cos\Lambda}\;\frac{AB + 2\cos\Lambda}{A + 2\cos\Lambda}\left(\frac{C_{l_\beta}}{C_L}\right)_{M=0}$	(6)	6
$\left(\frac{C_{n_\beta}}{C_L^2}\right)_M$	$\frac{A + 4\cos\Lambda}{AB + 4\cos\Lambda}\;\frac{A^2B^2 + 4AB\cos\Lambda - 8\cos^2\Lambda}{A^2 + 4A\cos\Lambda - 8\cos^2\Lambda}\left(\frac{C_{n_\beta}}{C_L^2}\right)_{M=0}$	(8)	7
$\left(\frac{C_{Y_\beta}}{C_L^2}\right)_M$	$\frac{A + 4\cos\Lambda}{AB + 4\cos\Lambda}\left(\frac{C_{Y_\beta}}{C_L^2}\right)_{M=0}$	(9)	3
$\left(C_{L_q}\right)_M$	$\frac{A + 2\cos\Lambda}{AB + 2\cos\Lambda}\left(C_{L_q}\right)_{M=0}$	(10)	2
$\left(C_{m_q}\right)_M$	$\frac{\frac{A^3\tan^2\Lambda}{AB + 6\cos\Lambda} + \frac{3}{B}}{\frac{A^3\tan^2\Lambda}{A + 6\cos\Lambda} + 3}\left(C_{m_q}\right)_{M=0}$	(11)	8
$\left(\frac{C_{l_r}}{C_L}\right)_M$	$\frac{1 + \frac{A(1 - B^2)}{2B(AB + 2\cos\Lambda)} + \frac{AB + 2\cos\Lambda}{AB + 4\cos\Lambda}\;\frac{\tan^2\Lambda}{8}}{1 + \frac{A + 2\cos\Lambda}{A + 4\cos\Lambda}\;\frac{\tan^2\Lambda}{8}}\left(\frac{C_{l_r}}{C_L}\right)_{M=0}$	(15)	9

[1]The symbol B used in these equations represents $\sqrt{1 - M^2\cos^2\Lambda}$.

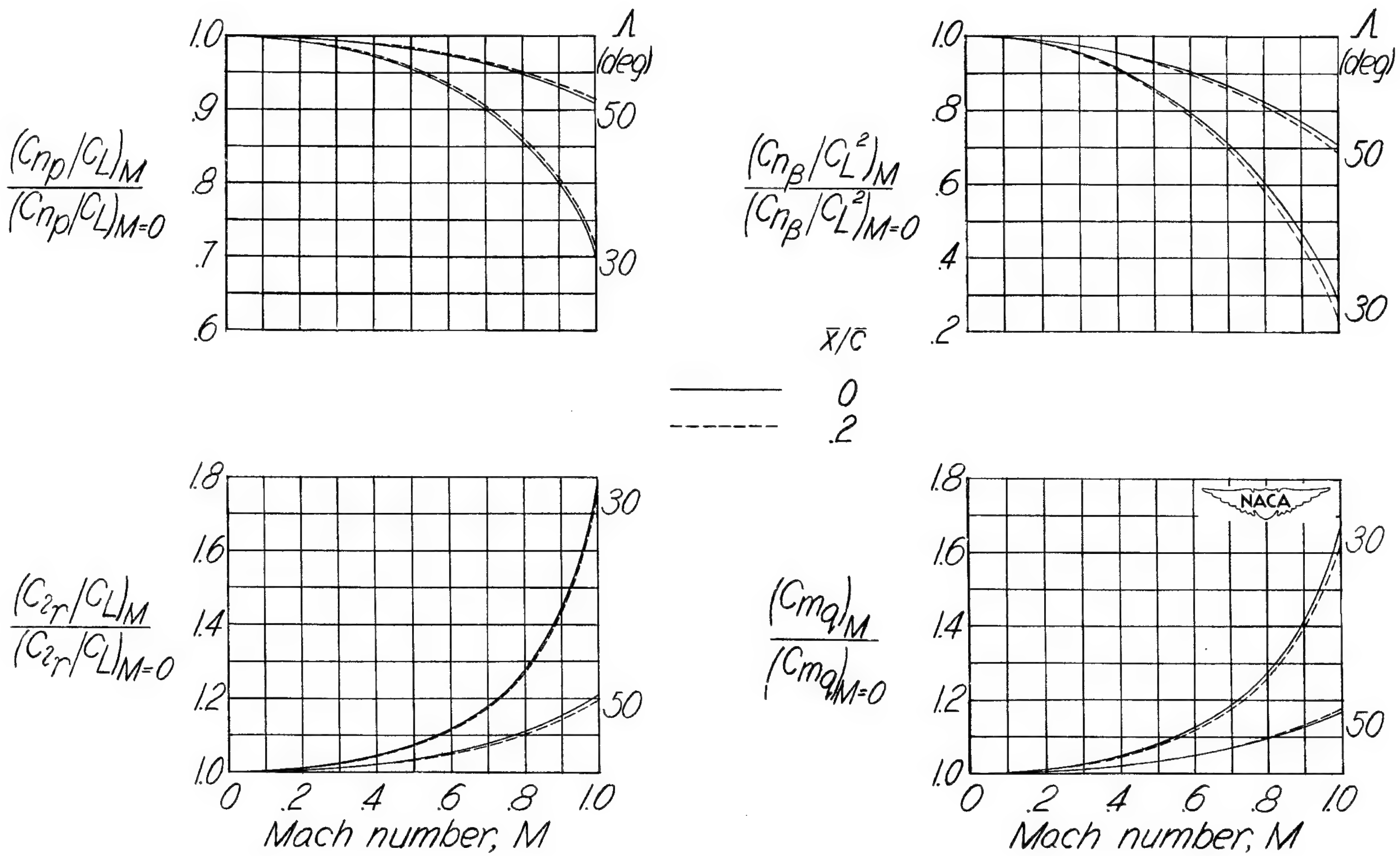

Figure 1.– The effect of the static margin $\bar{x}/\bar{c}$ on the variation of compressibility corrections with Mach number for a representative wing with two angles of sweepback. A = 4.

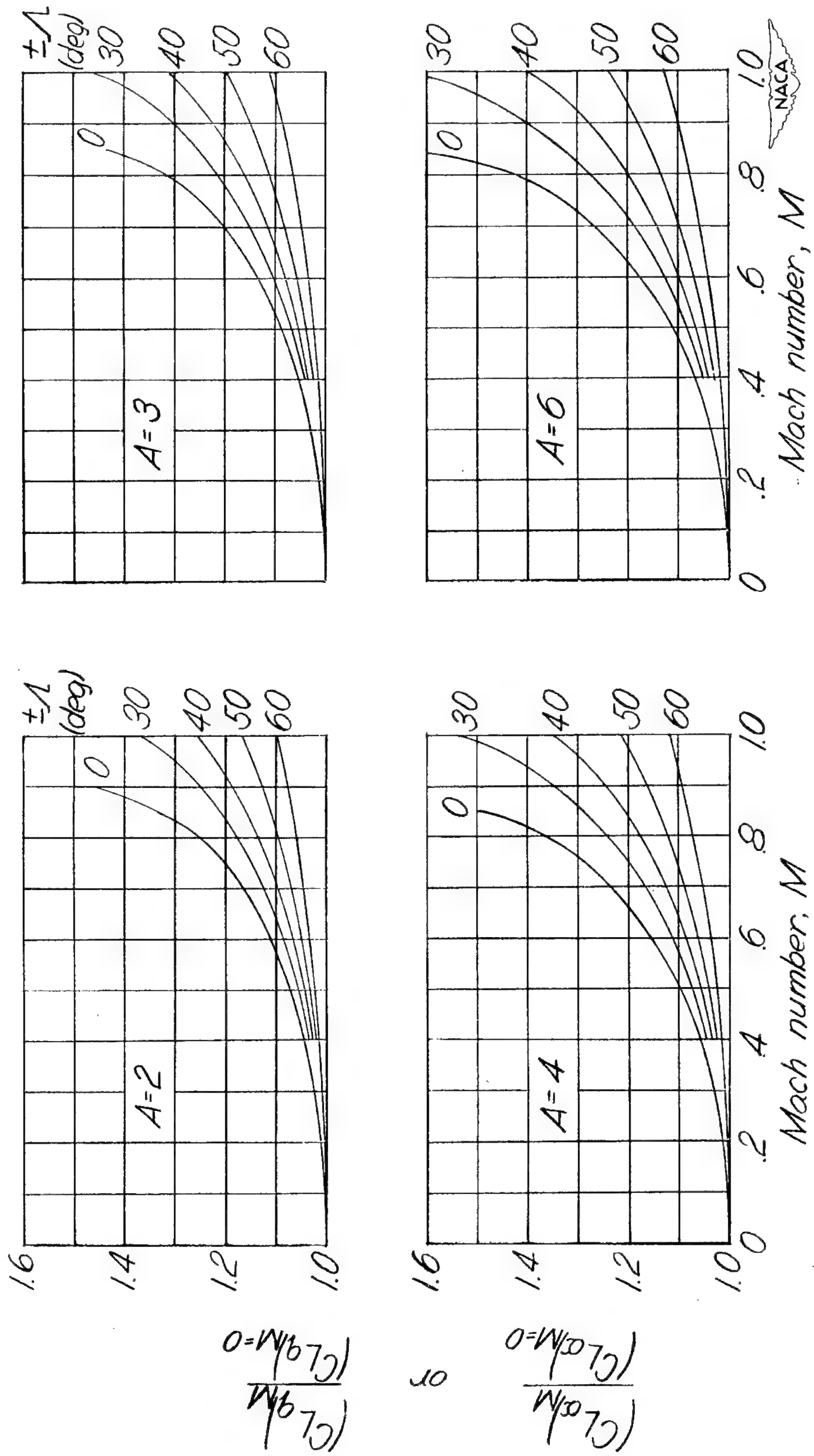

Figure 2.– Corrections for the effect of compressibility on the lift-curve slope, equation (1), and the lift due to pitching, equation (10).

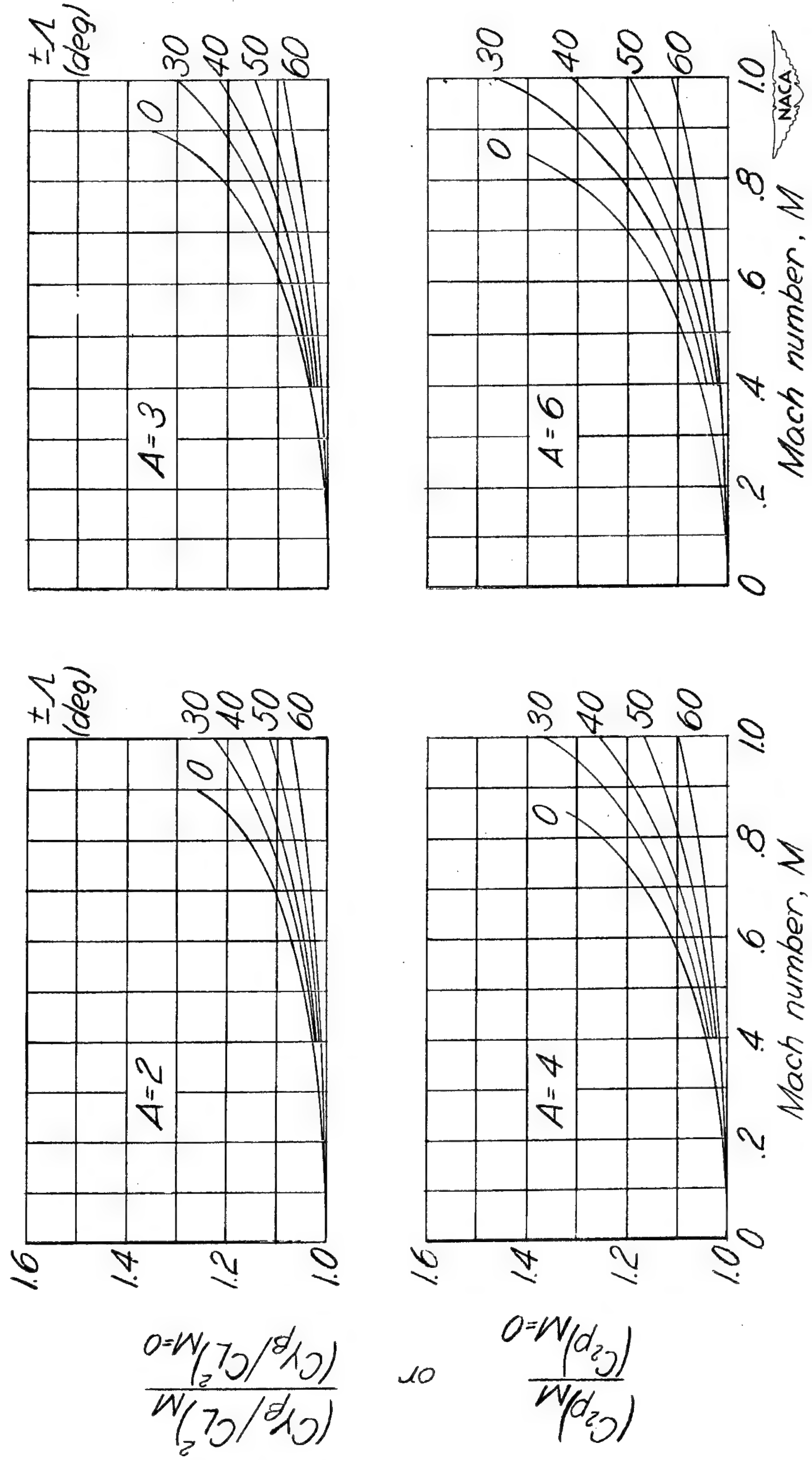

Figure 3.— Corrections for the effect of compressibility on the rolling moment due to rolling, equation (2), and the lateral force due to sideslip, equation (9).

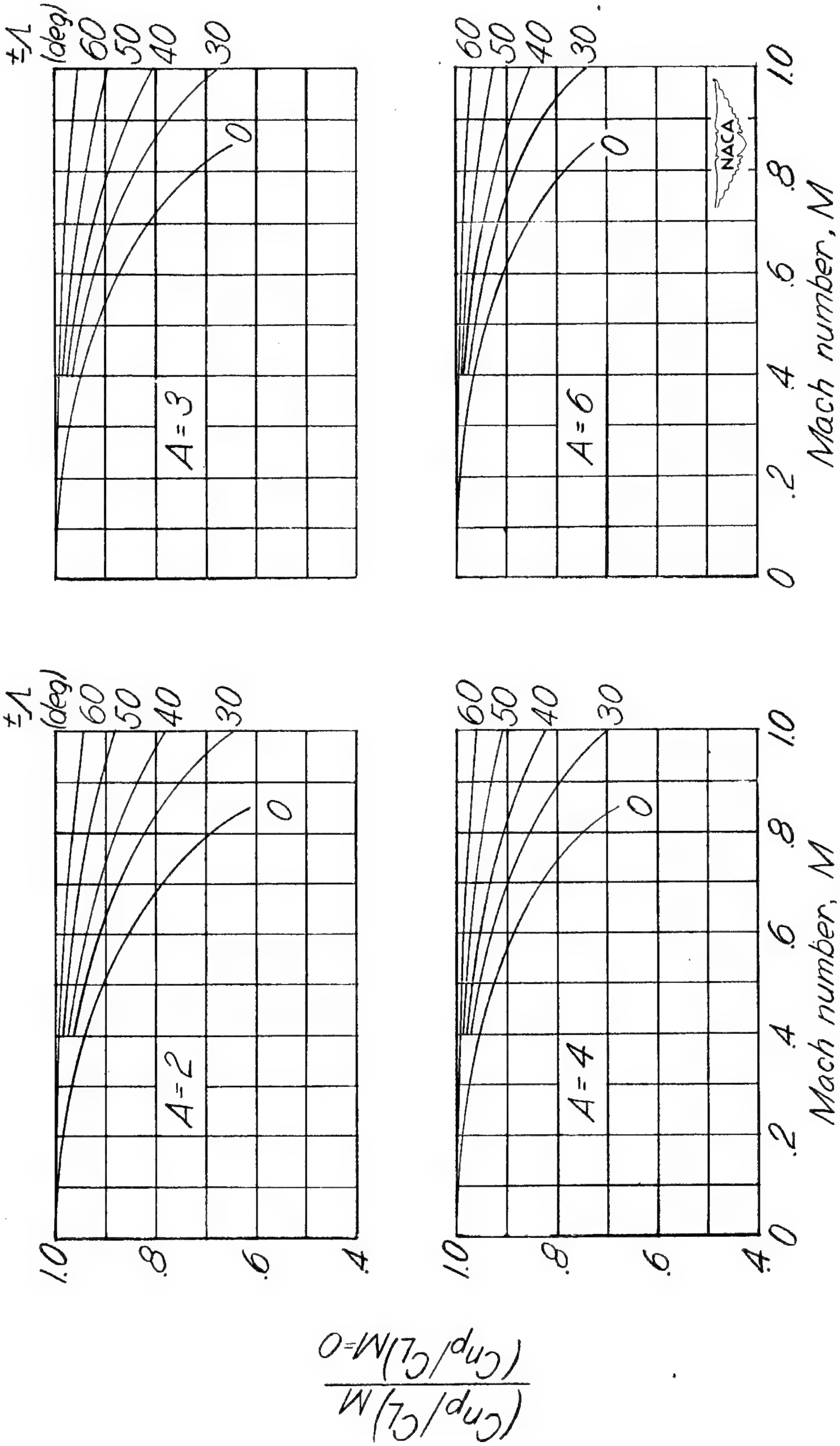

Figure 4.- Corrections for the effect of compressibility on the yawing moment due to rolling, equation (3). $\frac{\bar{x}}{c} = 0$.

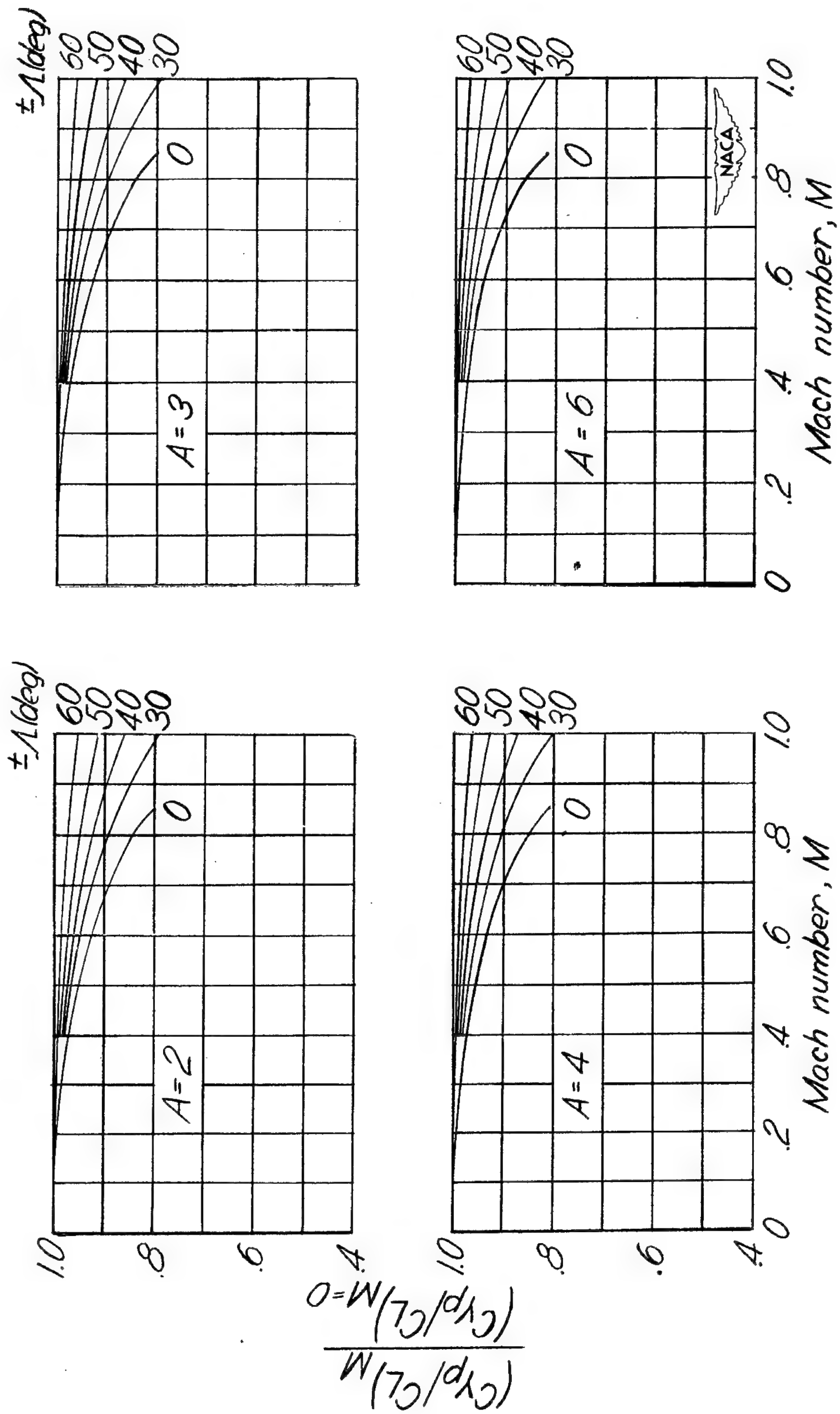

Figure 5.- Corrections for the effect of compressibility on the lateral force due to rolling, equation (4).

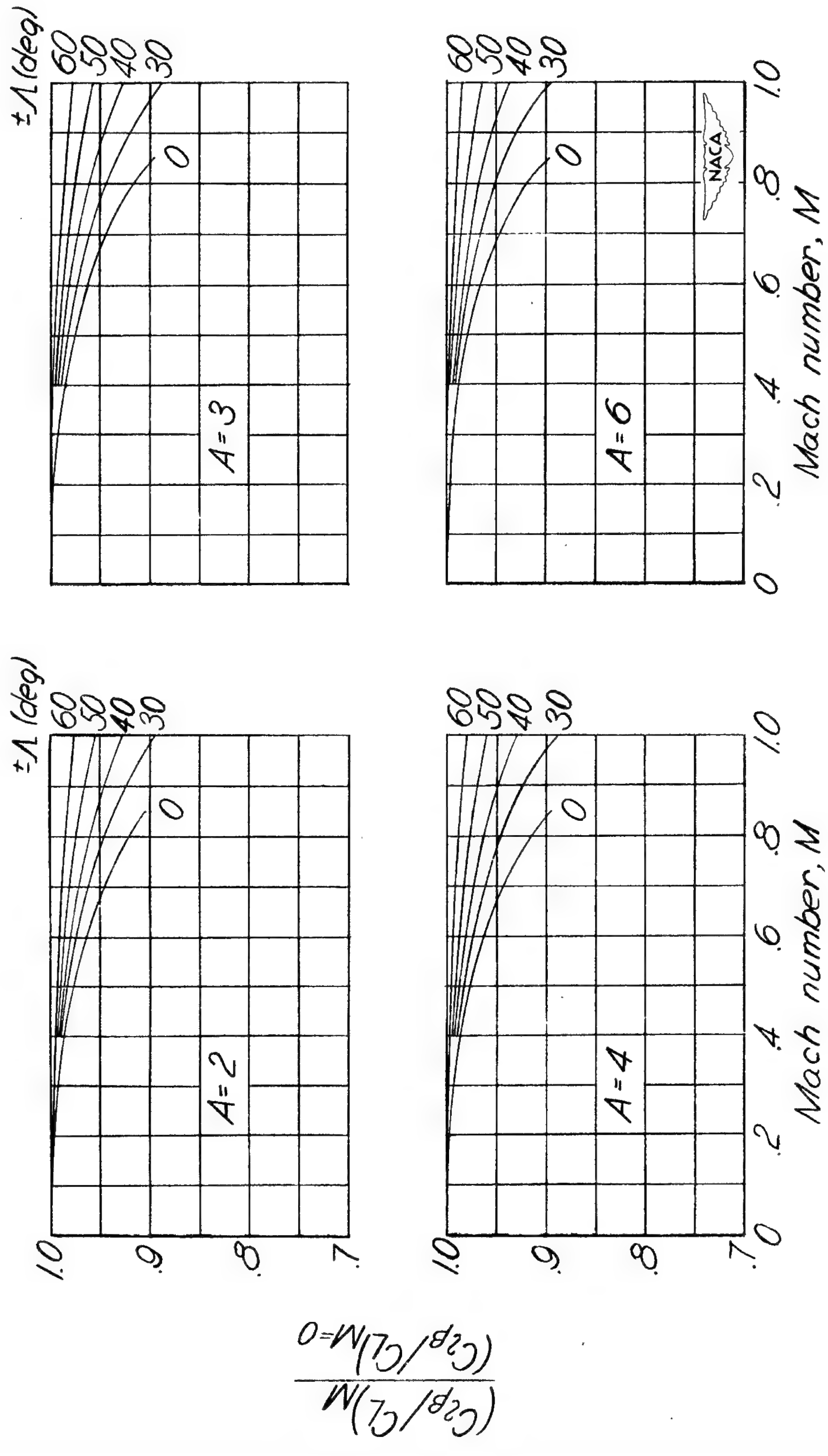

Figure 6.- Corrections for the effect of compressibility on the rolling moment due to sideslip, equation (6).

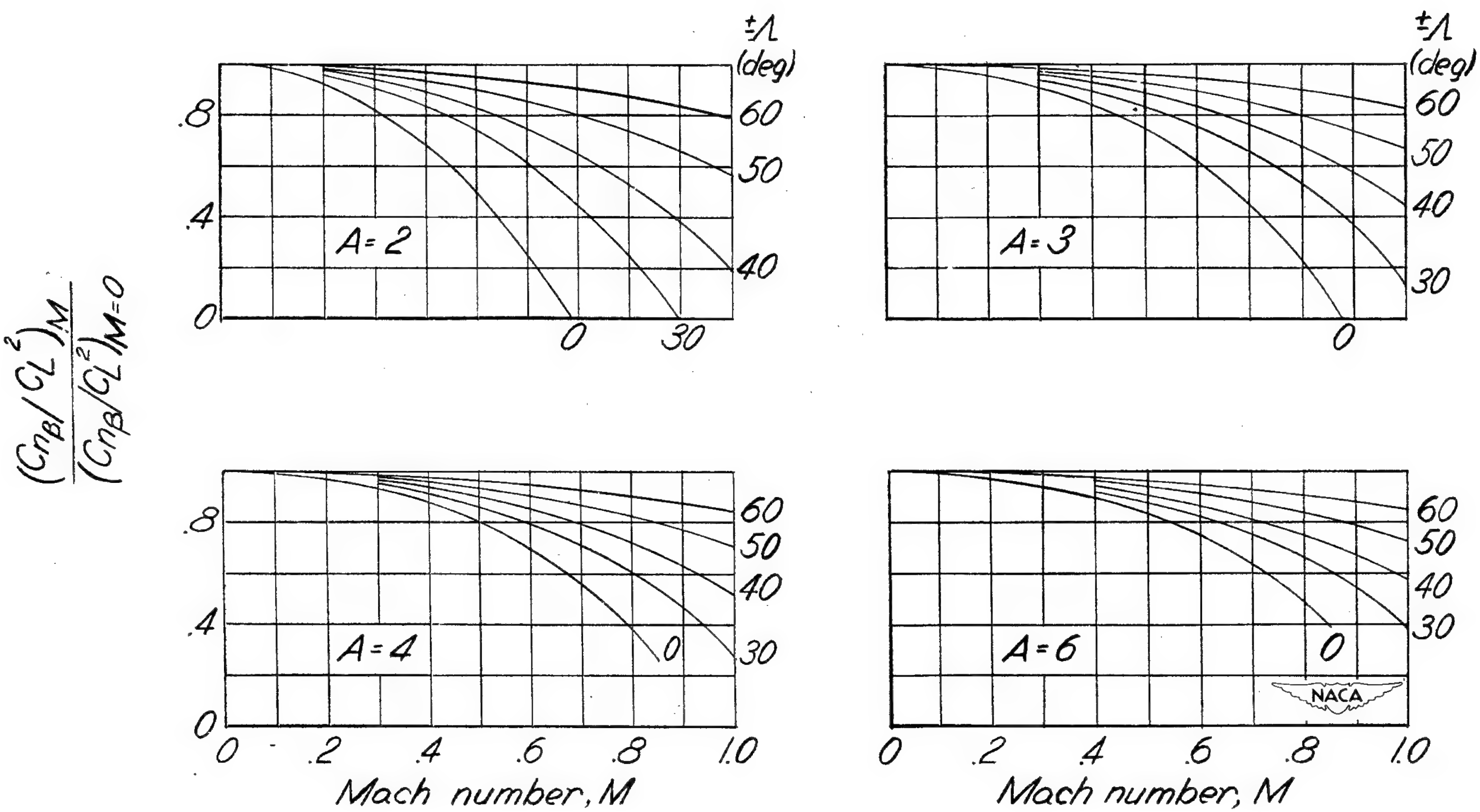

Figure 7.– Corrections for the effect of compressibility on the yawing moment due to sideslip, equation (8). $\frac{\bar{x}}{c} = 0$.

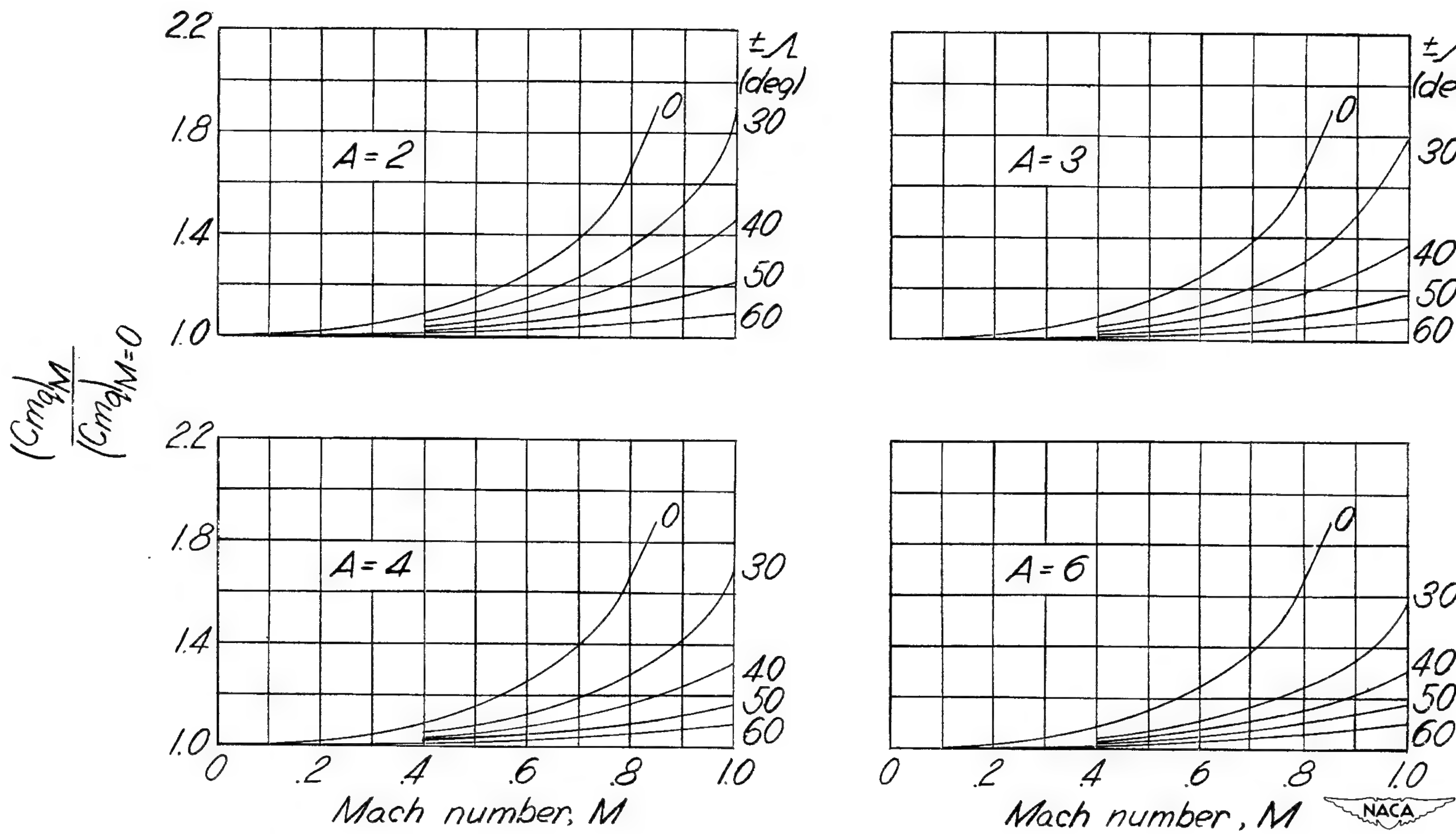

Figure 8.- Corrections for the effect of compressibility on the pitching moment due to pitching, equation (11). $\frac{\bar{x}}{c} = 0$.

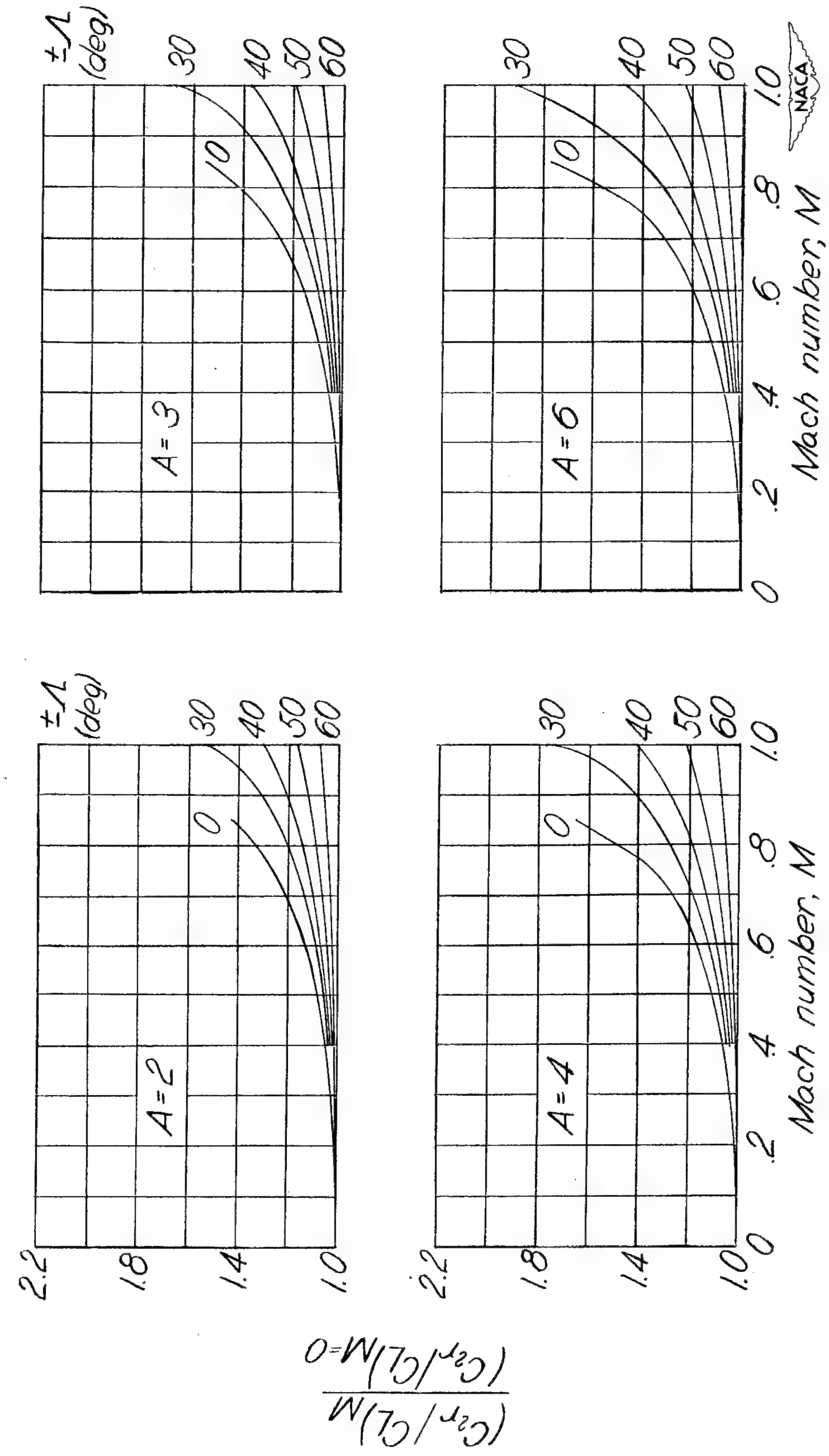

Figure 9.- Corrections for the effect of compressibility on the rolling moment due to yawing, equation (15). $\frac{\bar{x}}{c} = 0$.

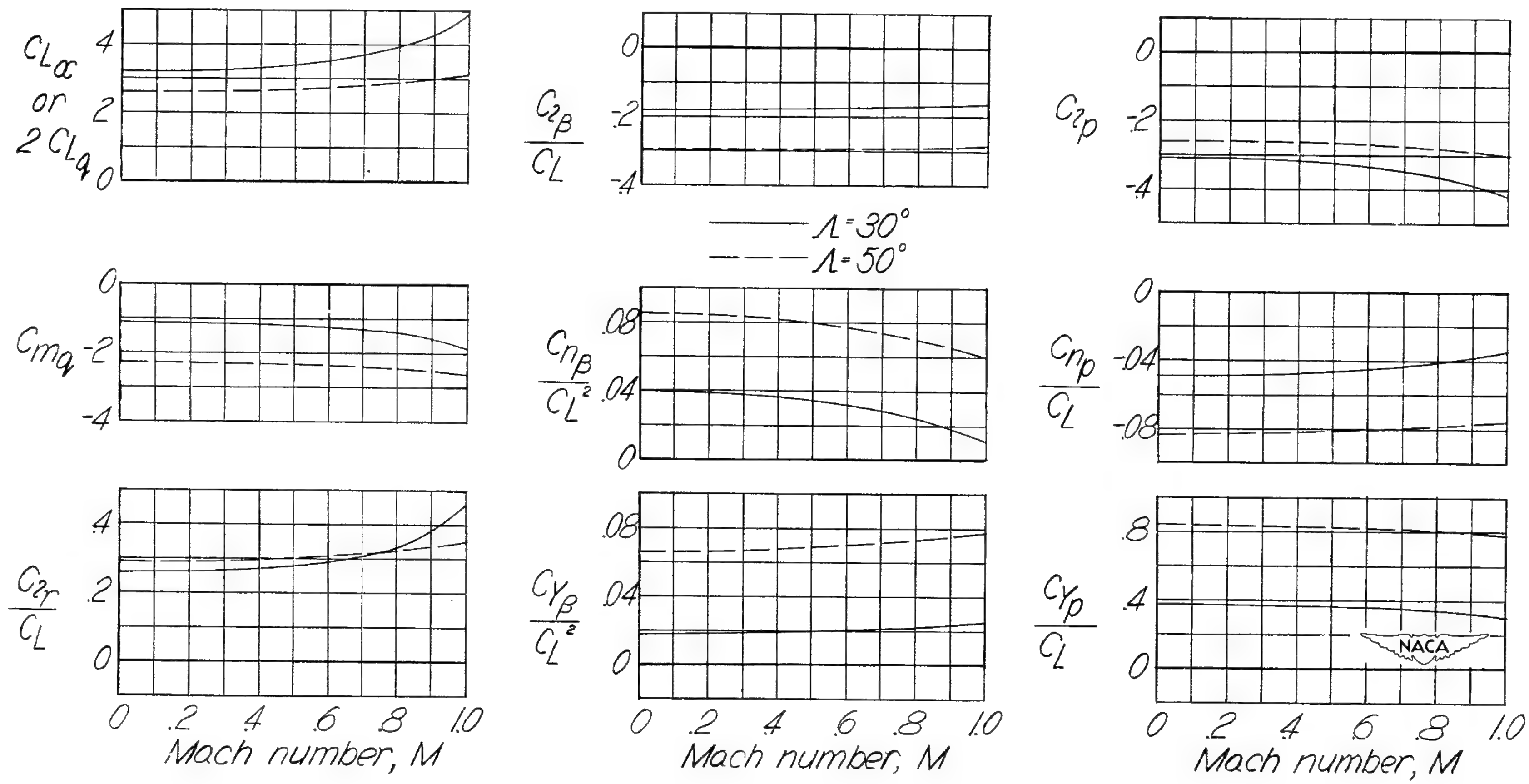

Figure 10.– Variation with Mach number of several stability derivatives of a representative wing for two angles of sweep. $A = 4$; $\frac{\bar{x}}{c} = 0$. (Incompressible-flow values are taken from reference 6.)

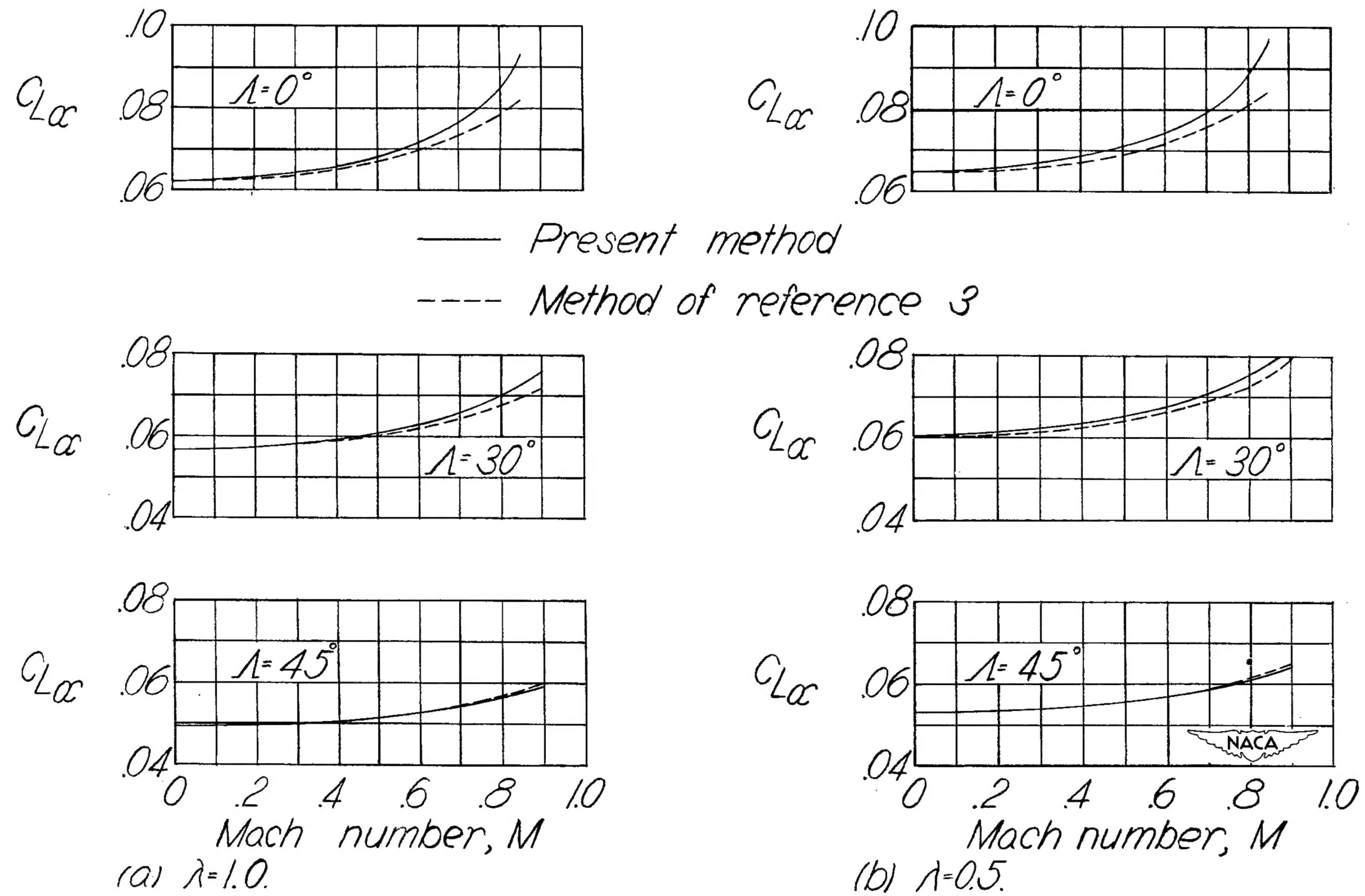

Figure 11.- Comparison of two methods for calculating Mach number effects on lift-curve slopes for two taper ratios. A = 4. (Incompressible-flow values calculated by method of Weissinger in reference 3.)

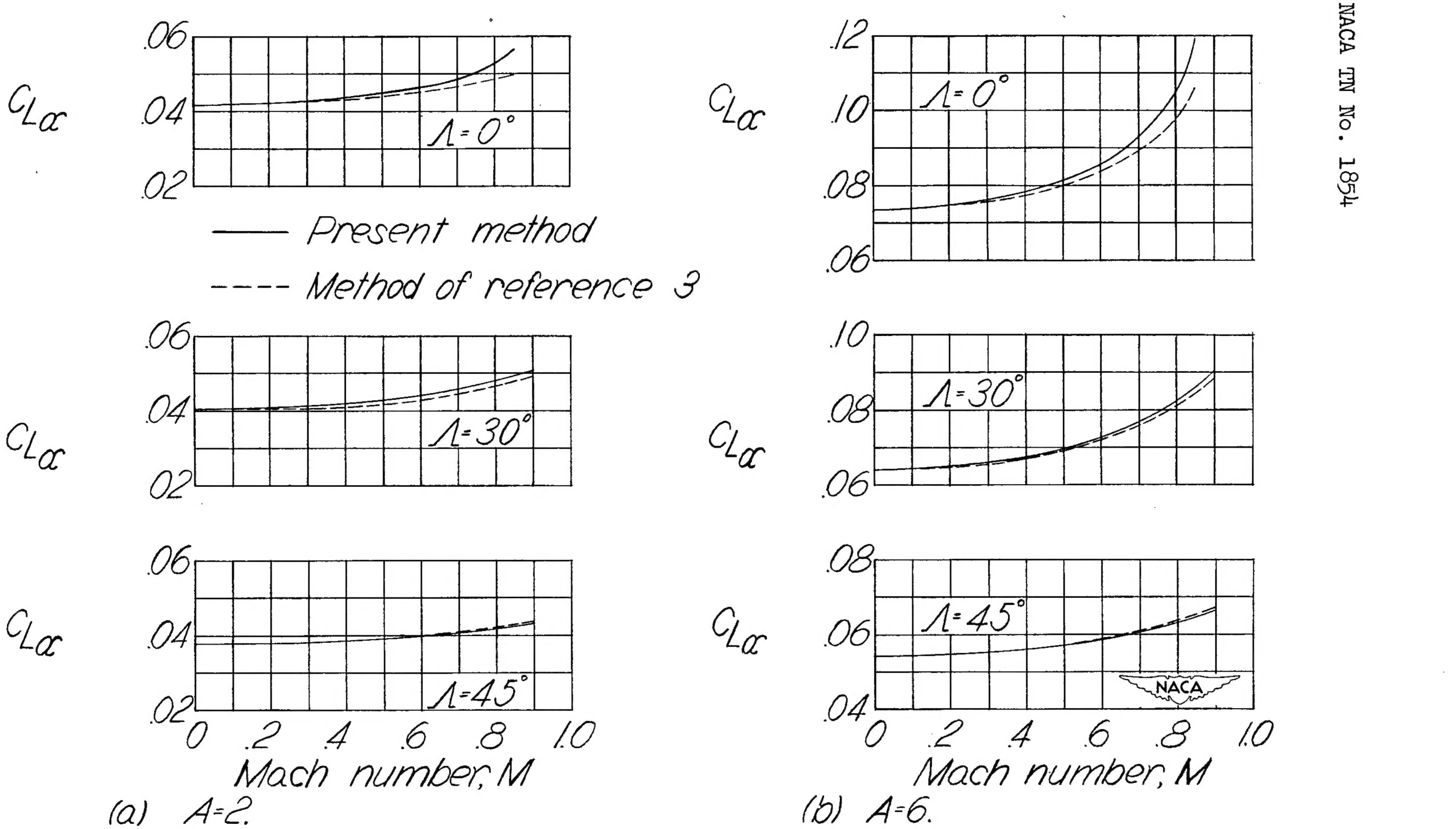

Figure 12.- Comparison of two methods for calculating Mach number effects on lift-curve slopes for two aspect ratios. $\lambda = 1.0$. (Incompressible-flow values calculated by method of Weissinger in reference 3.)

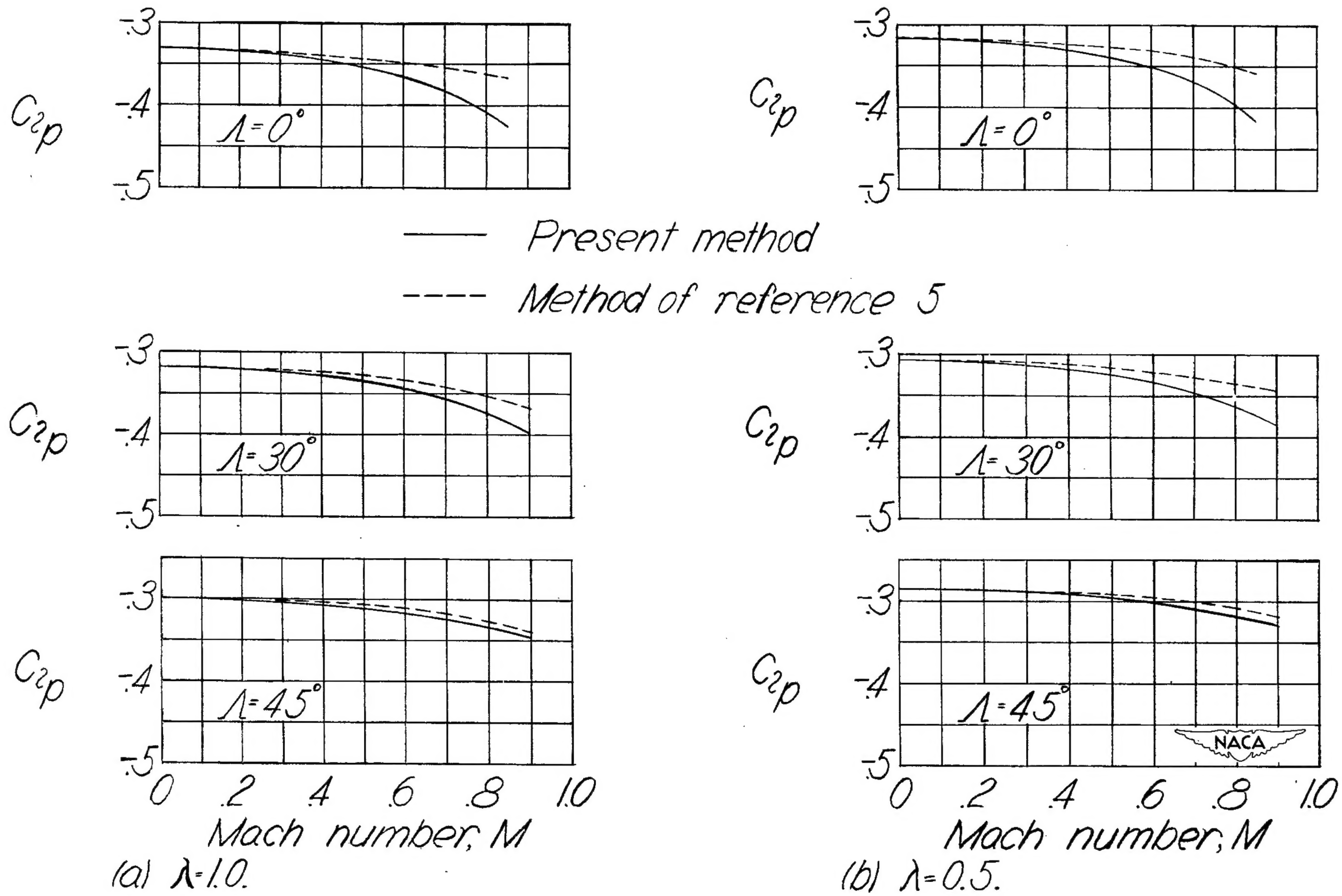

Figure 13.– Comparison of two methods for calculating Mach number effects on damping in roll for two taper ratios. A = 4. (Incompressible–flow values calculated by method of Weissinger in reference 5.)

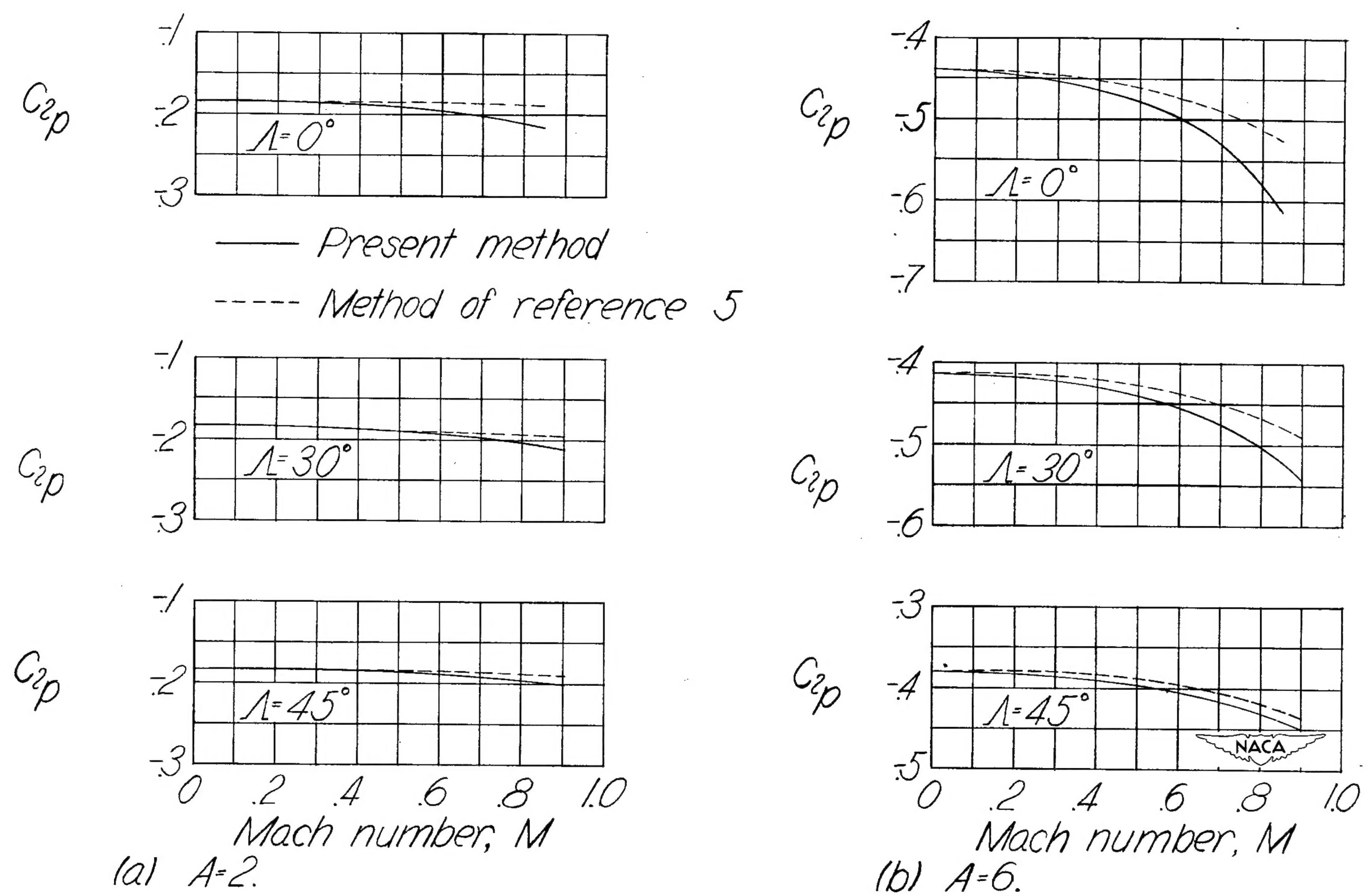

Figure 14.– Comparison of two methods for calculating Mach number effects on damping in roll for two aspect ratios. $\lambda = 1.0$. (Incompressible–flow values calculated by method of Weissinger in reference 5.)